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Globally Regular Model of the Electron in General Relativity

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Necessary conditions on a reasonable description of a physical object are suggested. The globally regular solutions of Petrov type D of the Einstein–Maxwell field equations and their generation solutions are derived for a charged perfect fluid sphere. A globally regular model of a stationary electron is established in the framework of general relativity. The quantitative relations between the inertial and the electromagnetic mass of an electron and between the electron mass and radius are explained.

1. INTRODUCTION

For a long time, people have looked for a complete theory which describes the behavior of an electron. From Maxwell–Lorentz theory to Mie's and Weyl's theories, they all tried to explain the electromagnetic origin of mass in a classical sense, but faced difficulties. For example, a nonelectromagnetic force is needed to overcome the electrostatic repulsion of the charge and to prevent the electron from "exploding" (Jackson, 1975; Pauli, 1958, p. 184).

Quantum electrodynamics has had great success in describing the electron–photon interaction, but artificial suppositions and techniques such as renormalization must be used to explain the finite electron mass (Fukuda *et al.*, 1949). The origin of the finite electron mass is a puzzling problem in both classical electromagnetic theory and quantum theory.

Since the 1970s some authors (Horwitz and Katz, 1971; Katz and Horwitz, 1971) have made new tries in the framework of general relativity.

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In particular, Tiwari *et al.* (1985) and Gautreau (1985) (TRKG) have shown the possibility of constructing pure electromagnetic mass models of the electron in the framework of general relativity. It becomes a pressing task to check the obtained conclusion and explore other possible gravitational models of an electron.

In the present work, a criterion for a reasonable description is given first. A possible globally regular model of the electron is then discussed using a Heintzmann-like technique (Kramer *et al.*, 1980).

2. PHYSICALLY REASONABLE DESCRIPTION

Any reasonable description of a physical object should satisfy the following requirements:

1. The field source which is used to describe an object must be physically reasonable, i.e., the energy-momentum tensor of the field source must satisfy the dominant energy conditions given by Hawking and Ellis (1973).
2. The metric coefficients are free from singularities except for trivial coordinate singularities.
3. The whole space-time of the model is causally well behaved and smoothly matched.

The above requirements are actually the conditions for globally regular solutions given by Shen and Zhu (1985). Thus we suggest that the necessary conditions to provide a reasonable description for a physical object in the framework of general relativity should be that the Einstein field equations associated with the model can give globally regular solutions. In accordance with this criterion, the model given by Israel (1970) is not a physical realistic model and the model given by TRKG can be accepted.

3. GLOBALLY REGULAR SOLUTIONS OF PETROV TYPE D

In the preceding section, we pointed out that the particular solutions obtained by TRKG are globally regular solutions. These solutions have no proper singularities and are smoothly matched on the boundary.

In this section we further consider globally regular solutions of Petrov type D for a charged perfect fluid sphere.

The energy-momentum tensor of a charged perfect fluid sphere is

$$T_{\lambda\nu} = \begin{cases} (p+\mu)v_\lambda v_\nu + pg_{\lambda\nu} + \dfrac{1}{4}\left(F_{\lambda\alpha}F_\nu{}^\alpha - \dfrac{1}{4}g_{\lambda\nu}F_{\alpha\beta}F^{\alpha\beta}\right) & (r \le r_b) \\ \dfrac{1}{4\pi}\left(F_{\lambda\alpha}F_\nu{}^\alpha - \dfrac{1}{4}g_{\lambda\nu}F_{\alpha\beta}F^{\alpha\beta}\right) & (r \ge r_b) \end{cases} \tag{1}$$

The Einstein–Maxwell field equations take the form

$$R_{\lambda\nu} - \frac{1}{2} g_{\lambda\nu} R = 8\pi T_{\lambda\nu} \tag{2}$$

$$F^{\lambda\nu}{}_{;\nu} = 4\pi J^{\lambda} \tag{3}$$

$$F_{\lambda\nu;\rho} + F_{\rho\lambda;\nu} + F_{\nu\rho;\lambda} = 0 \tag{4}$$

with

$$J^{\lambda} = \rho_e V^{\lambda} \tag{5}$$

$$V^0 = (-g_{00})^{-1/2} \tag{6}$$

$$V^i = 0 \tag{7}$$

$$V^{\nu} V_{\nu} = -1 \tag{8}$$

where μ, p, and r_b are the mass density, the pressure, and the radius of the charged perfect fluid sphere, respectively.

The exterior metric ($r \geq r_b$) is the Reissner–Nordström metric,

$$ds^2 = -\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right) dt^2 + \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta \, d\phi^2) \tag{9}$$

The coordinate r used in equation (9) has a clear physical meaning. By a measurement of the physical length of a great circle we can determine the value of the coordinate r for the sphere considered. The metric is of Petrov type D. Here M and Q are the total mass and the total charge of the sphere, respectively.

The interior metric is assumed to also be of Petrov type D and have the form

$$ds^2 = -e^{2\nu} dt^2 + e^{2\lambda} dr^2 + r^2(d\theta^2 + \sin^2\theta \, d\phi^2) \tag{10}$$

where

$$e^{2\nu} = e^{-2\lambda} \tag{11}$$

Then the Einstein–Maxwell field equations ($r \leq r_b$) can be reduced to

$$\frac{d}{(-g)^{1/2} dr} [(-g)^{1/2} F^{01}] = 4\pi\rho_e e^{-\nu} \tag{12}$$

$$e^{-2\lambda}\left(\frac{1}{r^2} - \frac{2\lambda'}{r}\right) - \frac{1}{r^2} = -8\pi\mu - E^2 \tag{13}$$

$$\frac{1}{r^2} - e^{-2\lambda}\left(\frac{1}{r^2} + \frac{2\nu'}{r}\right) = -8\pi p + E^2 \tag{14}$$

$$e^{-2\lambda}\left[\nu'\lambda' - \nu'^2 - \nu'' - \left(\frac{\nu' - \lambda'}{r}\right)\right] = -8\pi p - E^2 \tag{15}$$

From equations (11)–(15), we obtain

$$e^{2\nu} = e^{-2\lambda} = 1 - \frac{2[m^{(0)}(r) + m^{(e)}(r)]}{r} + \frac{Q^2}{r^2} \tag{16}$$

$$p = -\mu \tag{17}$$

$$\frac{dE^2(r)}{dr} + \frac{4E^2(r)}{r} = -8\pi\mu' \tag{18}$$

with

$$E^2(r) = -F_{01}F^{01} = \frac{Q^2}{r^4} \tag{19}$$

$$F(r) = \int E^2(r)r^2\, dr \tag{20}$$

$$Q(r) = 4\pi \int_0^r \rho_e^* r^2\, dr \tag{21}$$

$$m^{(0)}(r) = 4\pi \int_0^r \mu r^2\, dr \tag{22}$$

$$m^{(e)}(r) = \frac{1}{2}\left[\frac{Q^2}{r} + F(r)\right] \tag{23}$$

$$M(r_b) = m^{(0)}(r_b) + m^{(e)}(r_b) \tag{24}$$

where $\rho_e^* = \rho_e e^{\lambda}$ is the proper charge density, $m^{(0)}(r)$ is the pure gravitational mass, $m^{(e)}(r)$ is the electromagnetic self-energy mass, and $M(r)$ is the total mass.

As seen from equation (18), the electric field is closely related to the density of the perfect fluid. For any given reasonable charge or mass distributions, we can find the globally regular solutions.

We assume that $\mu = Ar^{\alpha} + B$, where A and B are constants. Thus equations (16)–(23) give

$$E^2(r) = \frac{1}{r^4}\left[\int -8\pi\mu' r^4\, dr + C\right] = -\frac{8\pi A\alpha r^{\alpha}}{\alpha + 4} + \frac{C}{r^4} \tag{25}$$

$$Q^2(r) = E^2(r)r^4 = -\frac{8\pi A\alpha r^{\alpha+4}}{\alpha + 4} + C \tag{26}$$

$$F(r) = \int E^2(r) r^2 \, dr = -\frac{8\pi A\alpha r^{\alpha+3}}{(\alpha+4)(\alpha+3)} - \frac{C}{r} + D \tag{27}$$

$$m^{(0)}(r) = 4\pi \int_0^r \mu r^2 \, dr = \frac{4\pi A r^{\alpha+3}}{\alpha+3} + \frac{4\pi B r^3}{3} \tag{28}$$

$$m^{(e)}(r) = \frac{1}{2}\left[\frac{Q^2(r)}{r} + F(r)\right] = -\frac{4\pi A\alpha r^{\alpha+3}}{\alpha+3} + \frac{D}{2} \tag{29}$$

$$p = -\mu = -(Ar^\alpha + B) \tag{30}$$

$$e^{2\nu} = e^{-2\lambda} = 1 - \frac{32\pi A r^{\alpha+2}}{(\alpha+4)(\alpha+3)} - \frac{8\pi B r^2}{3} + \frac{C}{r^2} - \frac{D}{r} \tag{31}$$

Here C and D are integration constants.

The regularities demand

$$C = D = 0$$

and

$$\alpha \geq 0$$

From equation (21), we obtain the proper charge density

$$\rho_e^* = (-A\alpha)^{1/2}(8\pi)^{-1/2}(\alpha+4)^{1/2} r^{(\alpha/2)-1} \tag{32}$$

The regularity of the charge density at $r = 0$ further limits α to

$$\alpha \geq 2$$

The junction conditions at $r = r_b$ give

$$A = -\frac{(\alpha+4)Q^2}{8\pi\alpha r_b^{\alpha+4}} \tag{33}$$

$$B = -Ar_b^\alpha = \frac{(\alpha+4)Q^2}{8\pi\alpha r_b^4} \tag{34}$$

Then we have

$$p = -\mu = -\frac{(\alpha+4)Q^2}{8\pi r_b^4}\left[1 - \left(\frac{r}{r_b}\right)^\alpha\right] \tag{35}$$

$$\rho_e^* = \frac{(\alpha+4)Q r^{(\alpha/2)-1}}{8\pi r_b^{(\alpha/2)+2}} \tag{36}$$

$$E^2 = \frac{Q^2 r^2}{r_b^6} \tag{37}$$

$$e^{2\nu} = e^{-2\lambda} = 1 - \frac{(\alpha+4)Q^2r^2}{3\alpha r_b^4} + \frac{4Q^2r^{\alpha+2}}{\alpha(\alpha+3)r_b^{\alpha+4}} \tag{38}$$

$$m^{(0)}(r) = \frac{(\alpha+4)Q^2r^3}{6\alpha r_b^4} - \frac{(\alpha+4)Q^2r^{\alpha+3}}{2\alpha(\alpha+3)r_b^{\alpha+4}} \tag{39}$$

$$m^{(e)}(r) = \frac{(\alpha+4)Q^2r^{\alpha+3}}{2(\alpha+3)r_b^{\alpha+4}} \tag{40}$$

The total mass of the charged perfect fluid sphere is given by

$$M(r) = m^{(0)} \quad + m^{(e)}(r) = \frac{(\alpha+4)Q^2r^3}{6\alpha r_b^4} - \frac{(\alpha-1)(\alpha+4)Q^2r^{\alpha+3}}{2\alpha(\alpha+3)r_b^{\alpha+4}} \tag{41}$$

Substituting $r = r_b$ into equations (39)–(41) yields

$$m^{(0)}(r_b) = \frac{(\alpha+4)Q^2}{6(\alpha+3)r_b} \tag{42}$$

$$m^{(e)}(r_b) = \frac{(\alpha+4)Q^2}{2(\alpha+3)r_b} \tag{43}$$

$$M(r_b) = \frac{2(\alpha+4)Q^2}{3(\alpha+3)r_b} \tag{44}$$

If M and Q represent the total mass m_e and charge e of an electron, respectively, then equations (42)–(44) mean

$$r_b = \frac{2(\alpha+4)}{3(\alpha+3)} r_e \tag{45}$$

where $r_e = e^2/m_e$ is the classical electron radius.

For $\alpha \geq 2$, the radius of the charged sphere with the mass distribution $\mu = Ar^\alpha + B$ satisfies the following inequality:

$$\frac{4}{5} r_e \geq r_b \geq \frac{2}{3} r_e$$

In addition, we see from equations (42)–(44)

$$m^{(0)} = \frac{1}{4} M = \frac{1}{4} m_e \tag{46}$$

$$m^{(e)} = \frac{3}{4} M = \frac{3}{4} m_e \tag{47}$$

which are independent of α. The case of $\alpha = 2$ gives the results for the uniform charge density distribution obtained by Tiwari *et al.* (1985). Similarly, for other reasonable charge or mass distribution, we can easily find globally regular solutions of Petrov type D.

4. GENERATION SOLUTIONS

So far, the obtained solutions are all of Petrov type D. The equation of state for matter is $p = -\mu$. Are there globally regular solutions of other type? We discuss them with the help of a generation technique.

If we introduce the new variables

$$x = r^2$$

$$y = e^{\nu}$$

$$w = (1 - e^{-2\lambda})/2x$$

then for the regular charged perfect fluid sphere, the Einstein–Maxwell field equations (12)–(15) are

$$8\pi\mu + E^2 = 6w + 4xw_{,x} \tag{48}$$

$$8\pi p - E^2 = -2w + 4(1 - 2xw)y_{,x}y^{-1} \tag{49}$$

with

$$(2xy_{,x} + y)w_{,x} + (2y_{,x} + 4xy_{,xx})w = 2y_{,xx} - x^{-1}yE^2 \tag{50}$$

When $E^2 = 0$, equations (48)–(50) reduce to those given by Heintzmann (Kramer *et al.*, 1980).

Quantities with a caret correspond to the new solution and quantities without a caret correspond to the old solution. Adopting the Heintzmann generation technique, we have the new solution

$$\hat{y} = y = e^{\nu} \tag{51}$$

$$\hat{w} = w_0 + \beta w_H \tag{52}$$

$$\hat{e}^{-2\lambda} = e^{-2\lambda} - 2\beta x w_H \tag{53}$$

$$8\pi\hat{p} - \hat{E}^2 = 8\pi p - E^2 - 2\beta(1 + 4xy_{,x}y^{-1})w_H \tag{54}$$

$$8\pi\hat{\mu} + E^2 = 8\pi\mu + E^2 + 2\beta(3w_H + 2xw_{H,x} \tag{55}$$

$$\hat{E}^2 = E^2 - \beta[(2xy_{,x}y^{-1} + 4x^2y_{,xx}y^{-1})w_H + (2x^2y_{,x}y^{-1} + x)w_{H,x})] \tag{56}$$

with

$$E^2 = 2xy_{,xx}y^{-1} - (2xy_{,x}y^{-1} + 4x^2y_{,xx}y^{-1})w_0 - (2x^2y_{,x}y^{-1} + x)w_{0,x} \tag{57}$$

$$8\pi p - E^2 = 4(1 - 2xw_0)y_{,x}y^{-1} - 2w_0 \tag{58}$$

$$8\pi\mu + E^2 = 6w_0 + 4xw_{0,x} \tag{59}$$

$$w_H = (2xy_{,x} + y)^{-2} \exp\left[4\int (2xy_{,x} + y)^{-1} y_{,x}\, dx\right] \tag{60}$$

For ease of representation, we discuss the case with the mass distribution $\mu = Ar^{\alpha} + B$ ($\alpha = 2$). The old solution is

$$y = e^{\nu} = \left(1 - \frac{625\sigma}{256}\hat{x} + \frac{3125\sigma}{2048}\hat{x}^2\right)^{1/2} \tag{61}$$

$$e^{-2\lambda} = 1 - 2xw_0 \tag{62}$$

$$w_0 = \left(\frac{625}{512} - \frac{3125}{4096}\hat{x}\right)\frac{\sigma}{x_e} \tag{63}$$

$$8\pi\mu = -8\pi p = \left(\frac{1875}{256} - \frac{46{,}875}{4096}\hat{x}\right)\frac{\sigma}{x_e} \tag{64}$$

$$E^2 = \frac{15{,}625}{4096}\cdot\frac{\sigma}{x_e}\hat{x} \tag{65}$$

Here $\sigma = e^2/x_e$, $\hat{x} = x/x_e$, $x_e = r_e^2$, $x_b = r_b^2$, and $r_b = \frac{4}{5}r_e$. Then generalization of the above solution is

$$\hat{y} = y = \left(1 - \frac{625\sigma}{256}\hat{x} + \frac{3125\sigma}{2048}\hat{x}^2\right)^{1/2} \tag{66}$$

$$\hat{E}^2 = E^2 = \frac{15{,}625\sigma}{4096x_e}\hat{x} \tag{67}$$

$$\hat{e}^{-2\lambda} = e^{-2\lambda} - 2\beta x w_H \tag{68}$$

$$8\pi\hat{\mu} = 8\pi\mu + 2\beta(3w_H + 2xw_{H,x}) \tag{69}$$

$$\begin{aligned} 8\pi\hat{p} &= 8\pi p - 2\beta(1 + 4xy_{,x}y^{-1})w_H \\ &= -8\pi\hat{\mu} + \beta[(4 - 8xy_{,x}y^{-1})w_H + 4xw_{H,x}] \end{aligned} \tag{70}$$

where

$$\begin{aligned} w_H = &\left(1 - \frac{625\sigma}{256}\hat{x} + \frac{3125\sigma}{2048}\hat{x}^2\right)\left(1 - \frac{625\sigma}{128}\hat{x} + \frac{9375\sigma}{2048}\hat{x}^2\right)^{-4/3} \\ &\times \exp\left\{-\frac{2}{3}\left(\frac{96}{125\sigma} - 1\right)^{-1/2} \operatorname{arctg}\left[\left(\frac{15}{8}\hat{x} - 1\right)\right.\right. \\ &\left.\left.\times\left(\frac{96}{125\sigma} - 1\right)^{-1/2}\right]\right\} \end{aligned} \tag{71}$$

$$W_{H,x} = \frac{625\sigma}{256x_e}\left(1 - \frac{15}{4}\hat{x} + \frac{9375}{2048}\sigma\hat{x}^2 - \frac{15{,}625\sigma}{8192}\hat{x}^3\right)$$
$$\times\left(1 - \frac{625\sigma}{128}\hat{x} + \frac{9375\sigma}{2048}\hat{x}^2\right)^{-7/3}\exp\left\{-\frac{2}{3}\left(\frac{96}{125\sigma} - 1\right)^{-1/2}\right.$$
$$\left.\times\ \mathrm{arctg}\left[\left(\frac{15}{8}\hat{x} - 1\right)\left(\frac{96}{125\sigma} - 1\right)^{-1/2}\right]\right\} \tag{72}$$

Application of the boundary condition $\hat{p} = 0$ at $x = x_b$ gives

$$\beta = \left(\frac{46{,}875\sigma}{8192x_e}\eta - \frac{1875\sigma}{512x_e}\right)\left(1 - \frac{625}{128}\sigma\eta + \frac{9375\sigma}{2048}\eta^2\right)^{4/3}$$
$$\times\left(1 - \frac{1875\sigma\eta}{256} + \frac{15{,}625\sigma}{2048}\eta^2\right)\exp\left\{\frac{2}{3}\left(\frac{96}{125\sigma} - 1\right)^{-1/2}\right.$$
$$\left.\times\ \mathrm{arctg}\left[\left(\frac{15\eta}{8} - 1\right)\left(\frac{96}{125\sigma} - 1\right)^{-1/2}\right]\right\} \tag{73}$$

where $\eta = x_b/x_e$ is a free parameter. For $r_b = \frac{4}{5}r_e$, we have $\eta = 16/25$ and $\beta = 0$. The new solution is simply the old solution given by Tiwari *et al.* (1985).

Because $\sigma = e^2/x_e = 2.395 \times 10^{-43} \ll 1$ for an electron, equation (73) is approximately reduced to

$$\beta \simeq \left(\frac{46{,}875\eta}{8192} - \frac{1875}{512}\right)\frac{\sigma}{x_e} \tag{74}$$

Obviously, $\beta > 0$ for $\eta > 16/25$ (i.e., $r_b > \frac{4}{5}r_e$), and $\beta < 0$ for $\eta < 16/25$.

A physically reasonable energy-momentum tensor should obey the dominant energy conditions

$$\hat{\mu} \geq 0 \tag{75}$$

$$-\hat{\mu} \leq \hat{p} \leq \hat{\mu} \tag{76}$$

From equations (69), (71), (72), and (74), we see that

$$8\pi\hat{\mu} = 8\pi\mu + 2\beta(3w_H + 2xw_{H,x}) \simeq 8\pi\mu + 6\beta$$
$$\geq 8\pi\mu \geq 0$$

and the condition

$$-8\pi\hat{\mu} \leq 8\pi\hat{p} \leq 8\pi\hat{\mu}$$

is equivalent to

$$-8\pi\hat{\mu} \leq -8\pi\hat{\mu} + 4\beta \leq 8\pi\mu + 6\beta$$

which is automatically satisfied for $\beta > 0$. Therefore, the dominant energy conditions (75) and (76) hold. The obtained new solution is physically reasonable according to the criteria given in Section 2.

In the new solution, the equation of state is

$$\hat{p} = -\hat{\mu} + \frac{\beta}{8\pi}[(4 - 8xy_{,x}y^{-1})W_H + 4xw_{H,x}] \tag{77}$$

It is seen that if the radius r_b of the source is such that $r_b > \frac{4}{5}r_e$, there is another possible model relevant to the idea of Lorentz in which the pressure $\hat{p}$, the mass density $\hat{\mu}$, and the constant β depend on charge e^2. And the equation of state is no longer $p = -\mu$.

5. DISCUSSION

A possible new pure-charge model has been obtained by using a Heintzmann-like generation technique.

As some authors have pointed out, the electron's mass is associated with the Schwarzschild gravitational mass given by general relativity (Tiwari *et al.*, 1985):

$$m_e = 4\pi \int_0^r \mu r^2\, dr + \frac{1}{2}\int_0^r E^2 r^2\, dr + \frac{e^2}{2r}$$

In order to understand correctly the origin of electron mass, we believe that the total mass should be divided into two parts: one of them is contributed from the inertial mass

$$m^{(0)}(r) = 4\pi \int_0^r \mu r^2\, dr$$

and the other from the electromagnetic field and its source

$$m^{(e)}(r) = \frac{1}{2}\int_0^r E^2 r^2\, dr + \frac{e^2}{2r}$$

The calculation of Section 3 has shown that $m^{(0)} = \frac{1}{4}m_e$ and $m^{(e)} = \frac{3}{4}m_e$ for the equation of state $p = -\mu$. The result agrees with the conclusion that the total energy of a stationary electron is equal to four-thirds of its electromagnetic energy given by Lorentz (Pauli, 1958, p. 185).

A rough estimation of the inertial mass and electromagnetic self-energy mass for the new generation solution of Section 4 gives

$$m^{(0)} = 4\pi \int_0^{r_b} \hat{\mu} r^2\, dr = 4\pi \int_0^{r_b} \mu r^2\, dr + 3\beta \int_0^{r_b} r^2\, dr$$
$$\simeq \frac{37{,}500e^2}{8192r_e}\eta^{5/2} - \frac{20{,}000e^2}{8192r_e}\eta^{3/2}$$

$$m^{(e)} = \frac{1}{2}\int_0^{r_b} \hat{E}^2 r^2\, dr + \frac{e^2}{2r_b}$$
$$= \frac{1}{2}\int_0^{r_b} E^2 r^2\, dr + \frac{e^2}{2r_b}$$
$$= \frac{3125e^2}{8192r_e}\eta^{5/2} + \frac{e^2}{r_e}\eta^{-1/2}$$
$$M = m^{(0)} + m^{(e)}$$
$$= \frac{40{,}625e^2}{8192r_e}\eta^{5/2} - \frac{20{,}000e^2}{8192r_e}\eta^{3/2} + \frac{e^2}{r_e}\eta^{-1/2}$$

The total mass M and the ratio of the inertial mass to the electromagnetic energy will vary with the free parameter $\eta = (r_b/r_e)^2$.

It is noteworthy that the dependence of the inertial mass on the charge should only be regarded as a quantitative relation between the inertial and the electromagnetic mass rather than as the physical basis of the pure electromagnetic nature of electron mass.

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